Preface

This volume is the Pre-conference Proceedings of the Second International Conference on Algebra and Discrete Mathematics (ICADM-2020) conducted by the Department of Mathematics, DDE, Madurai Kamaraj University during June 24 -26, 2020 in online mode. The main themes of the conference are Algebra, Discrete Mathematics and their applications. The role of Algebra and Discrete Mathematics in the field of Mathematics has been rapidly increasing over several decades. In recent decades, the graphs constructed out of algebraic structures have been extensively studied by many authors and have become a major field of research. The benefit of studying these graphs is that one may find some algebraic property of the under lying algebraic structure through the graph property and the vice-versa. The tools of each have been used in the other to explore and investigate the problem in deep. This conference is organized with the aim of providing an avenue for discussing recent advancements in these fields and exploring the possibility of effective interactions between these two areas.

The aim of the conference is to introduce research topics in the main streams of Algebra and Discrete Mathematics to young researchers especially research students, and encourage them to collaborate in teams lead by well-known mathematicians from various countries.

This volume makes available a record of articles presented in the conference. This volume contains the papers presented in the conference without any referring process.

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DECOMPOSITION OF JUMP GRAPH OF PATHS

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Abstract

The Jump graph $J(G)$ of a graph G is the graph whose vertices are edges of G and two vertices of $J(G)$ are adjacent if and only if they are not adjacent in G. Equivalently complement of line graph $L(G)$ is the Jump graph $J(G)$ of G. In this paper, we give necessary and sufficient condition for the decomposition of Jump graph of paths into various graphs such as paths, cycles, stars, complete graphs and complete bipartite graphs.

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1 Introduction

Let $G = (V, E)$ be a simple undirected graph without loops or multiple edges. A *path* on n vertices is denoted by P_n, *cycle* on n vertices is denoted by C_n and *complete graph* on n vertices is denoted by K_n. The *neighbourhood* of a vertex v in G is the set $N(v)$ consisting of all vertices that are adjacent to v. $|N(v)|$ is called the degree of v and is denoted by $d(v)$. A *complete bipartite graph* with partite sets V_1 and V_2, where $|V_1| = r$ and $|V_2| = s$, is denoted by $K_{r,s}$. The graph $K_{1,r}$ is called a *star* and is denoted by S_r. *Claw* is a star with three edges. For any set S of points of G, *induced subgraph* $< S >$ is the maximal subgraph of G with point set S. The terms not defined here are used in the sense of [2].

A *decomposition* of a graph G is a family of edge-disjoint subgraphs $\{G_1, G_2, \ldots, G_k\}$ such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_k)$. If each G_i is isomorphic to H for some subgraph H of G, then the decomposition is called a H-decomposition of G.

The *Jump graph* $J(G)$ of a graph G is the graph whose vertices are edges of G and two vertices of $J(G)$ are adjacent if and only if they are not adjacent in G. This concept was introduced by Chartrand in [1]. Let $J(P_n)$ denote the Jump graph of paths. Then $J(P_n)$ is a connected graph if and only if $n \geq 5$. Let us consider the connected jump graph of paths. Let the edges of path P_n be labelled as $x_1, x_2, \ldots, x_{n-1}$. Then the vertices of $J(P_n)$ be labelled as $x_1, x_2, \ldots, x_{n-1}$. Since the number of edges of path P_n is $(n-1)$, the number of vertices of $J(P_n)$ is $(n-1)$. The number of edges of Jump graph of paths $J(P_n)$ is $\binom{n-2}{2}$.

In 2010, Tay - Woei Shyu [6] gave necessary and sufficient condition for the decomposition of complete graph into P_4 's and S_4 's. In this paper, we give necessary and sufficient condition for the decomposition of Jump graph of paths into various graphs such as paths, cycles, stars, complete graphs and complete bipartite graphs.

Theorem 1.1. *Let n be an odd positive integer with $p = \frac{n-3}{2}$ and $q = \frac{(n-5)(n-3)}{8}$. There exists a decomposition of $J(P_n)$ into p copies of P_4 and q copies of C_4 iff $n \geq 5$ and $3p + 4q = \binom{n-2}{2}$.*

Proof. (Necessity) Let n be an odd positive integer. Suppose that there exists a decomposition of $J(P_n)$ into p copies of P_4 and q copies of C_4 where $p = \frac{n-3}{2}$ and $q = \frac{(n-5)(n-3)}{8}$. Clearly Jump graph of path $J(P_n)$ is a connected graph if and only if $n \geq 5$. Since n is odd, $n \geq 5$. Since $|E[J(P_n)]| = \binom{n-2}{2}$, we have $3p + 4q = \binom{n-2}{2}$.
(Sufficiency) Suppose $3p + 4q = \binom{n-2}{2}$ where $p = \frac{n-3}{2}$ and $q = \frac{(n-5)(n-3)}{8}$. Clearly $x_{2k-3}x_{2k-5}x_{2k-2}x_{2k-4}$; $3 \leq k \leq \frac{n+1}{2}$ forms P_4 in $J(P_n)$. Then we get $(\frac{n+1}{2} - 2)$ copies of P_4. Thus $p = \frac{n-3}{2}$. Also $\left\{x_1x_{2k-3}x_2x_{2k-2}x_1 / 4 \leq k \leq \frac{n+1}{2}\right\}$ $\cup$ $\left\{x_3x_{2k-3}x_4x_{2k-2}x_3 / 5 \leq k \leq \frac{n+1}{2}\right\}$ $\cup$ $\left\{x_5x_{2k-3}x_6x_{2k-2}x_5 / 6 \leq k \leq \frac{n+1}{2}\right\}$ $\cup \ldots \cup$ $\left\{x_{n-6}x_{2k-3}x_{n-5}x_{2k-2}x_{n-6} / k = \frac{n+1}{2}\right\}$ forms C_4 in $J(P_n)$. Then we get $\frac{(n-5)(n-3)}{8}$ copies of C_4. Therefore $q = \frac{(n-5)(n-3)}{8}$. Thus $E[J(P_n)] = \underbrace{E(P_4) \cup \ldots \cup E(P_4)}_{p\ times} \cup \underbrace{E(C_4) \cup \ldots \cup E(C_4)}_{q\ times}$ where $p = \frac{n-3}{2}$ and $q = \frac{(n-5)(n-3)}{8}$. Thus $J(P_n)$ is decomposable into p copies of P_4 and q copies of C_4. □

Theorem 1.2. *Let n be an even positive integer with $p = \frac{n-4}{2}$, $q = \frac{(n-6)(n-4)}{8}$ and $r = n - 3$. There exists a decomposition of $J(P_n)$ into p copies of P_4, q copies of C_4 and one copy of S_r iff $n \geq 6$ and $3p + 4q + r = \binom{n-2}{2}$.*

Proof. (Necessity) Let n be an even positive integer. Suppose that there exists a decomposition of $J(P_n)$ into p copies of P_4, q copies of C_4 and

one copy of S_r where $p = \frac{n-4}{2}$, $q = \frac{(n-6)(n-4)}{8}$ and $r = n-3$. Since $J(P_n)$ is connected, $n \geq 5$. Since n is even, $n \geq 6$. Since $|E[J(P_n)]| = \binom{n-2}{2}$, we have $3p + 4q + r = \binom{n-2}{2}$.
(Sufficiency) Consider $3p + 4q + r = \binom{n-2}{2}$ where $p = \frac{n-4}{2}$, $q = \frac{(n-6)(n-4)}{8}$ and $r = n-3$. Clearly $x_{2k-3}x_{2k-5}x_{2k-2}x_{2k-4}$; $3 \leq k \leq \frac{n}{2}$ forms P_4 in $J(P_n)$. Then we get $(\frac{n}{2} - 2)$ copies of P_4. Also the vertices $\{x_1x_{2k-3}x_2x_{2k-2}x_1/4 \leq k \leq \frac{n}{2}\} \cup \{x_3x_{2k-3}x_4x_{2k-2}x_3/5 \leq k \leq \frac{n}{2}\} \cup \{x_5x_{2k-3}x_6x_{2k-2}x_5/6 \leq k \leq \frac{n}{2}\} \cup \ldots \cup \{x_{n-7}x_{2k-3}x_{n-6}x_{2k-2}x_{n-7}/k = \frac{n}{2}\}$ forms C_4 in $J(P_n)$. Then we get $\frac{(n-6)(n-4)}{8}$ copies of C_4. Also the vertex x_{n-1} is not in any of the above P_4 and C_4. Since $d(x_{n-1}) = n-3$ in $J(P_n)$, x_{n-1} together with its neighbours forms S_{n-3}. Thus $E[J(P_n)] = \underbrace{E(P_4) \cup \ldots \cup E(P_4)}_{p\ times} \cup \underbrace{E(C_4) \cup \ldots \cup E(C_4)}_{q\ times} \cup E(S_r)$ where $p = \frac{n-3}{2}, q = \frac{(n-5)(n-3)}{8}$ and $r = n-3$. Thus $J(P_n)$ is decomposable into p copies of P_4, q copies of C_4 and one copy of S_r. □

Theorem 1.3. *Let n be an odd positive integer with $p = \frac{n-3}{2}$ and $q = \frac{n-5}{2}$. There exists a decomposition of $J(P_n)$ into p copies of P_4, q complete bipartite graphs of the form $K_{2,2l}$; $l = 1, 2, \ldots, \frac{n-5}{2}$ iff $n \geq 5$ and $3p + 2q(q+1) = \binom{n-2}{2}$.*

Proof. (Necessity) Given that there are p copies of P_4 and q complete bipartite graphs of the form $K_{2,2l}$; $l = 1, 2, \ldots, \frac{n-5}{2}$ where $p = \frac{n-3}{2}$ and $q = \frac{n-5}{2}$. Clearly $|E(J(P_n))| = \binom{n-2}{2}$. Thus we have $3p + 2q(q+1) = \binom{n-2}{2}$.
(Sufficiency) Consider $3p + 2q(q+1) = \binom{n-2}{2}$ where $p = \frac{n-3}{2}$ and $q = \frac{n-5}{2}$. Let the vertices of $J(P_n)$ be $x_1, x_2, \ldots, x_{n-1}$. Clearly $x_{2k-3}x_{2k-5}x_{2k-2}x_{2k-4}$; $3 \leq k \leq \frac{n+1}{2}$ forms P_4 in $J(P_n)$. Then we get $(\frac{n+1}{2} - 2)$ copies of P_4. Thus $p = (\frac{n+1}{2} - 2)$. Also, x_m and x_{m+1} are non adjacent vertices for $m = 1, 3, 5, \ldots, (n-6)$ and they are adjacent with each of the vertices $x_{m+4}, x_{m+5}, x_{m+6}, \ldots, x_{n-1}$. Thus we get $\frac{n-5}{2}$ complete bipartite graphs of the form $K_{2,2l}$; $l = 1, 2, \ldots, \frac{n-5}{2}$. Thus $E[J(P_n)] = \underbrace{E(P_4) \cup \ldots \cup E(P_4)}_{p\ times} \cup E(K_{2,2}) \cup E(K_{2,4}) \cup \ldots \cup E(K_{2,n-5})$ where $p = \frac{n-3}{2}$. Thus $J(P_n)$ is decomposable into p copies of P_4 and q complete bipartite graphs of the form $K_{2,2l}$; $l = 1, 2, \ldots, \frac{n-5}{2}$ where $p = \frac{n-3}{2}$ and $q = \frac{n-5}{2}$. □

Theorem 1.4. *Let n be an even positive integer with $p = \frac{n-4}{2}$ and $q = \frac{n-6}{2}$. There exists a decomposition of $J(P_n)$ into p copies of P_4, q complete bipartite graphs of the form $K_{2,2l+1}$; $l = 1, 2, \ldots, \frac{n-6}{2}$ and one claw iff $n \geq 6$ and $3p + 2q(q+2) + 3 = \binom{n-2}{2}$.*

Proof. (Necessity) Consider that there are p copies of P_4, q complete bipartite graphs of the form $K_{2,2l+1}$; $l = 1, 2, \ldots, \frac{n-6}{2}$ and one claw where $p = \frac{n-4}{2}$ and $q = \frac{n-6}{2}$. Since n is even and connected, we have $n \geq 6$ Clearly $|E[J(P_n)]| = \binom{n-2}{2}$. Thus we have $3p + 2q(q+2) + 3 = \binom{n-2}{2}$.

(Sufficiency) Consider $3p+2q(q+2)+3=\binom{n-2}{2}$ where $p=\frac{n-4}{2}$ and $q=\frac{n-6}{2}$. Let the vertices of $J(P_n)$ be $x_1,x_2,\ldots,x_{n-1}$. Clearly $x_{2k-3}x_{2k-5}x_{2k-2}x_{2k-4}$; $3\leq k\leq\frac{n}{2}$ forms P_4 in $J(P_n)$. Then we get $(\frac{n}{2}-2)$ copies of P_4. Thus $p=(\frac{n}{2}-2)$. Also, $\{x_m,x_{m+1}\}$ are non adjacent vertices where $m=1,3,5,\ldots,(n-7)$. Then they are adjacent with each of the vertices $x_{m+4},x_{m+5},x_{m+6},\ldots,x_{n-1}$. Thus we get $\frac{n-6}{2}$ complete bipartite graphs of the form $K_{2,2l+1}$; $l=1,2,\ldots,\frac{n-6}{2}$. Therefore $q=\frac{n-6}{2}$. Also x_{n-1} is not a vertex of any P_4 and $d(x_{n-1})=n-6$ in complete bipartite graph $K_{2,2l+1}$; $l=1,2,\ldots,\frac{n-6}{2}$. Since $d(x_{n-1})=n-3$ in $J(P_n)$, the remaining neighbours of x_{n-1} together with x_{n-1} forms a claw. Thus $E[J(P_n)]=\underbrace{E(P_4)\cup\ldots\cup E(P_4)}_{p\ times}$ $\cup E(K_{2,3})\cup E(K_{2,5})\cup\ldots\cup E(K_{2,n-5})\cup E(S_3)$ where $p=\frac{n-4}{2}$. Thus $J(P_n)$ is decomposable into p copies of P_4, q complete bipartite graphs of the form $K_{2,2l}$; $l=1,2,\ldots,\frac{n-6}{2}$ and one claw where $p=\frac{n-4}{2}$ and $q=\frac{n-6}{2}$. □

Theorem 1.5. *Let n be an even positive integer with $p=\frac{n-4}{2}$, $q=\frac{n-6}{2}$ and $r=\frac{n}{2}$. There exists a decomposition of $J(P_n)$ into two copies of S_p; $(\frac{n}{2}-2)$ copies of S_q and two complete graphs of the form K_r and K_{r-1} iff $n\geq 6$ and $2p-2q+(r-1)^2=\binom{n-2}{2}-\frac{nq}{2}$.*

Proof. (Necessity) We have $|E(J(P_n))|=\binom{n-2}{2}$. Since, there are two copies of S_p, $(\frac{n}{2}-2)$ copies of S_q and two complete graphs K_r and K_{r-1} where $p=\frac{n-4}{2}$, $q=\frac{n-6}{2}$ and $r=\frac{n}{2}$, we have $2p+(\frac{n}{2}-2)q+(r-1)^2=\binom{n-2}{2}$.
(Sufficiency) Consider $2p-2q+(r-1)^2=\binom{n-2}{2}-\frac{nq}{2}$. Let the vertices of $J(P_n)$ be labelled as $x_1,x_2,\ldots,x_{n-1}$. Now, the induced subgraphs $<\{x_1,x_3,\ldots,x_{\frac{n}{2}-1}\}>=K_{\frac{n}{2}}$ and $<\{x_2,x_4,\ldots,x_{\frac{n}{2}-2}\}>=K_{\frac{n}{2}-1}$. Let us partition $V(G)$ into V_1 and V_2 where $V_1=\{x_{1+2k}/k=0,1,\ldots,\frac{n-2}{2}\}$ and $V_2=\{x_{2+2k}/k=0,1,\ldots,\frac{n-4}{2}\}$. Consider $x_1,x_{n-1}\in V_1$. Clearly x_1 is not adjacent with x_2 and x_{n-1} is not adjacent with x_{n-2}. Also, both x_1 and x_{n-1} are adjacent with the remaining vertices in V_2. Therefore x_1 is adjacent with $(\frac{n}{2}-1)-1$ vertices. Similarly x_{n-1} is adjacent with $(\frac{n}{2}-1)-1$ vertices. Hence we get 2 copies of $S_{\frac{n-4}{2}}$. Therefore $p=\frac{n-4}{2}$. Each vertices of $V_1-\{x_1,x_{n-1}\}$ is adjacent with $(\frac{n}{2}-1)-2$ vertices in V_2. Thus $(\frac{n}{2}-2)$ vertices of V_1 is adjacent with $(\frac{n}{2}-3)$ vertices in V_2. Thus we have $(\frac{n}{2}-2)$ copies of $S_{\frac{n-6}{2}}$. Therefore $q=(\frac{n-6}{2})$. Thus $E[J(P_n)]=E(S_p)\cup E(S_p)\cup\underbrace{E(S_q)\cup E(S_q)\ldots\cup E(S_q)}_{(\frac{n-4}{2})\ copies}\cup E(K_r)\cup E(K_{r-1})$ where $p=\frac{n-4}{2}$, $q=\frac{n-6}{2}$ and $r=\frac{n}{2}$. Thus $J(P_n)$ is decomposable into two copies of S_p; $(\frac{n}{2}-2)$ copies of S_q and two complete graphs of the form K_r and K_{r-1}. □

Theorem 1.6. *Let n be an odd positive integer with $p=\frac{n-3}{2}$, $q=\frac{n-5}{2}$ and $r=\frac{n-1}{2}$. There exists a decomposition of $J(P_n)$ into one copy of S_p; $\frac{n-3}{2}$ copies of S_q and two copies of K_r iff $n\geq 5$ and $p-\frac{3q}{2}+r^2-r=\binom{n-2}{2}-\frac{nq}{2}$.*

Proof. (Necessity) We have $|E[J(P_n)]|=\binom{n-2}{2}$. Since there is one copy of

S_p, $\frac{n-3}{2}$ copies of S_q and two copies of K_r where $p = \frac{n-3}{2}$, $q = \frac{n-5}{2}$ and $r = \frac{n-1}{2}$, we have $p - \frac{3q}{2} + r^2 - r = \binom{n-2}{2} - \frac{nq}{2}$.
(Sufficiency) Suppose that $p - \frac{3q}{2} + r^2 - r = \binom{n-2}{2} - \frac{nq}{2}$ where $p = \frac{n-3}{2}$, $q = \frac{n-5}{2}$ and $r = \frac{n-1}{2}$. Let the vertices of $J(P_n)$ be labelled as $x_1, x_2, \ldots, x_{n-1}$. Now, the induced subgraphs $< \{x_1, x_3, \ldots, x_{\frac{n}{2}-1}\} > = K_{\frac{n-1}{2}}$ and $< \{x_2, x_4, \ldots, x_{\frac{n}{2}-2}\} > = K_{\frac{n-1}{2}}$. Let us partition $V(G)$ into V_1 and V_2 where $V_1 = \{x_{1+2k}/k = 0, 1, \ldots, \frac{n-3}{2}\}$ and $V_2 = \{x_{2+2k}/k = 0, 1, \ldots, \frac{n-3}{2}\}$. Let $x_1 \in V_1$. x_1 is not adjacent with only x_2 in V_2 and x_1 is adjacent with the remaining $\frac{n-1}{2} - 1$ vertices in V_2. Thus we get one copy of $S_{\frac{n-3}{2}}$. Therefore $p = \frac{n-3}{2}$. Each of the remaining vertices of $V_1 - \{x_1\}$ is adjacent with $(\frac{n-1}{2} - 2)$ vertices in V_2. Then we get $(\frac{n-3}{2})$ copies of $S_{\frac{n-5}{2}}$. Therefore $q = (\frac{n-5}{2})$. Thus $E[J(P_n)] = E(S_p) \cup \underbrace{E(S_q) \cup E(S_q) \cup \ldots \cup E(S_q)}_{(\frac{n-3}{2})\ copies} \cup E(K_r) \cup E(K_r)$ where $p = \frac{n-3}{2}$, $q = \frac{n-5}{2}$ and $r = \frac{n-1}{2}$. Thus $J(P_n)$ is decomposable into one copy of S_p; $\frac{n-3}{2}$ copies of S_q and two copies of K_r. □

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